Tribhuvan University

Institute of Science and Technology

2066

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Bachelor Level/ Second Year/ Third Semester/ Science — Full Marks: 60

Computer Science and Information Technology (CSc. 204) — Pass Marks: 24

(Numerical Method) — Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

1. Define the fixed-point iteration method. Given the function $f(x) = x^2 - 2x - 3 = 0$, rearrange the function in such a way that the iteration method converses to its roots. **(2+3+3)**
2. What do you mean by interpolation problem? Define divided difference table and construct the table from the following data set. **(2+2+4)**

X_i	3.2	2.7	1.0	4.8	5.6
F_i	22.0	17.8	14.2	38.3	51.7

OR

Find the least squares line that fits the following data.

X	1	2	3	4	5	6
Y	5.04	8.12	10.64	13.18	16.20	20.04

What do you mean by linear least square approximation?

3. Derive the composite formula for the trapezoidal rule with its geometrical figure. Evaluate $I = \int_0^1 e^{-x^2}\ dx$ using this rule with n=5, upto 6 decimal places. **(4+4)**
4. Solve the following system of algebraic linear equations using Jacobi or Gauss-Seidel iterative **(8)**

$$6x_1 - 2x_2 + x_3 = 11$$
$$-2x_1 + 7x_2 + 2x_3 = 5$$
$$X_1 + 2x_2 - 5x_3 = -1$$

5. Write an algorithm and computer program to fit a curve $y = ax^2 + bx + c$ for given sets of (x_i, y_i, g. 0 = 1, …, x) values by least square method. **(4+8)**
6. Derive a difference equation to represent a Poison's equation. Solve the Poison's equation $\nabla^2 f = 2x^2y^2$ over the domain $0 \le x \le 3, 0 \le y \le 3$ with $f = 0$ on the boundary and h = 1. **(3+5)**

7. Define ordinary differential equation of the first order. What do you mean by initial value problem? Find by Taylor's series method, the values of y at x = 0.1 and x = 0.2 to find places of decimal form

$\frac{dy}{dx} = x^2y - 1$, when y(0) = 1 **(2+6)**

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Attempt all questions:

1. Discuss methods of Half Interval and Newton's for solving the nonlinear equation f(x) = 0. Illustrate the methods by figure and compare them stating their advantages and disadvantages. **(8)**
2. Derive the equation for Lagrange's interpolating polynomial and find the value of f(x) at x = 1 for the following: **(4+4)**

X	-1	-2	2	4
F(x)	-1	-9	11	69

3. Write Newton-cotes integration formulas in basic form for x = 1, 2, 3 and give their composite rules. Evaluate $I = \int_{.2}^{1.5} e^{-x^2}\, dx$ using the Gaussian integration three point formula. **(4+4)**
4. Solve the following algebraic system of linear equations by Gauss-Jordan algorithm. **(8)**

$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 2 & 3 & 2 \\ 4 & -3 & 0 & 1 \\ 6 & 1 & -6 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -7 \\ 6 \end{bmatrix}$$

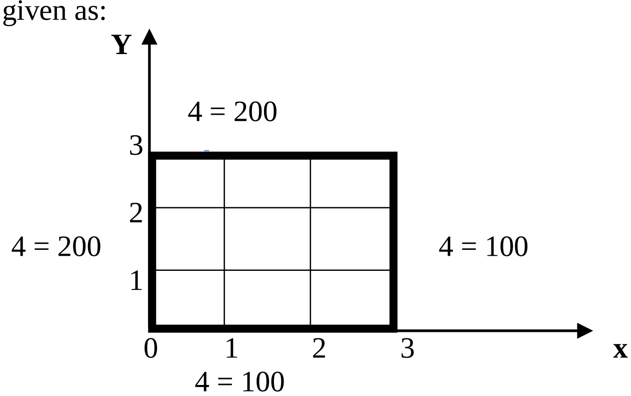
5. Write an algorithm and program to solve system of linear equations using Gauss-Seidel iterative method. **(4+8)**
6. Explain the Picard's proves of successive approximation. Obtain a solution upto the fifth approximation of the equation

$\frac{dy}{dx} = y + x$ such that y = 1 when x = 0

using Picard's process of successive approximations . **(2+6)**

7. Define a difference equation to represent a Laplace's equation. Solve the following Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ within $0 \le x \le 3, 0 \le y \le 3$

For the rectangular plate given as:



(3 + 5)

OR

Derive a difference equation to represent a Poison's equation. Solve the Poison's equation
$\nabla^2 f = 2x^2y^2$
Over the domain $0 \le x \le 3, 0 \le y \le 3$ with $f = 0$ on the boundary and h = 1. **(3+5)**

Prepared By ASCOL CSIT 2070 Batch

Tribhuvan University

Institute of Science and Technology

2068

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Bachelor Level/ Second Year/ Third Semester/ Science Full Marks: 60

Computer Science and Information Technology (CSc. 204) Pass Marks: 24

(Numerical Method) Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all questions:

1. Define the types of errors in numerical calculations. Derive the formula for secant method and illustrate the method by figure. **(4+4)**
2. Define the linear least squares approximations. Give the data set (x_i, y_i) as (20.5, 765), (32.7, 826), (51.0, 873), (73.2, 942), (95.7, 1032) find the linear least square to fit given data. **(2+6)**
3. Evaluate $I = \int_0^1 e^{-x^2}\ dx$ using trapezoidal rule with n=10. Also evaluate the same integral using Grossion 3 point formula and compare the result. **(4+4)**
4. Solve the following system of linear equations using Gauss-elimination method (use partial pivoting if necessary);

$$
\begin{aligned}
2x_2 + x_4 &= 0 \\
2x_1 + 2x_2 + 3x_3 + 2x_4 &= -2 \\
4x_1 - 3x_2 + x_4 &= -7 \\
6x_1 + x_2 - 6x_3 - 5x_4 &= 6
\end{aligned}
$$

(8)

OR

What do you mean by eigen -value eigen- vector problems? Find the largest eigen value correct to two significant digits and corresponding eigen vectors of the following matrix using power method.

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 1 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

(2+6)

5. Write an algorithm and program to solve system of linear equations using Gauss- Jordan method. **(4+8)**
6. Apply Runge-Kutta method of second order and fourth order to find an approximate value of y when x = 0.2 given that

$$\frac{\partial y}{\partial x} = x + y \ and\ y(0) = 1.$$

(8)

7. How can you solve Laplace's equation? Explain. The steady-state two dimensional heat flow in a metal plate is defined by $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0.$

   A steel plate of size 30 x 30cm is given. Two adjacent sides are placed at 100℃ and other side held at 0℃ . Find the temperature at interior points, assuming the grid size of 10 x 10cm. **(3+5)**

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(Numerical Method) — Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
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Attempt all questions:

1. Derive the formula to solve nonlinear equation using secant method. Using your formula estimate a real root of following nonlinear equation using secant method correct up to two decimal places $x^2 + \ln x = 3$. **(3+5)**
2. Estimate $f(3)$ from the following data using Cubic Spline interpolation.

x	1	2.5	4	5.7
f(x)	-2.0	4.2	14.4	31.2

OR

Find the best fitting quadratic polynomial from following data using least square approximation.

x	-2	-1.2	0	1	1.2	2.5	3	4.5	6.3
f(x)	10.39	2.96	-2.0	-2.63	-2.46	0.83	3.1	12.8	30.4

3. a) For the function $f(x) = e^x\sqrt{sinx + lnx}$ estimate $f'(6.3)$ and $f''(6.3)$ [take h = 0.01] **(4)**

   b) Evaluate $\int_1^2 (lnx + x^2\, sinx)dx$ using Gaussian integration 3 point formula. **(4)**

4. Solve the following set of equation using Gauss elimination or Gauss Jordan method

$$3x_1 + 5x_2 - 3x_3 + x_4 = 16$$
$$2x_1 + x_2 + x_3 + 4x_4 = 9$$
$$3x_1 - 4x_2 - x_4 = 1$$
$$2x_1 + x_2 - 3x_3 + 9x_4 = 5$$

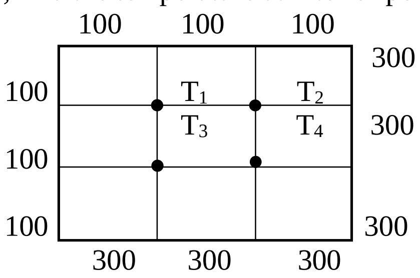
(8)

5. How can you solve higher order differential equation? Explain. Solve the following differential within $0 \le x \le 1$ using Heun's method.
$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2xy = 1$ with y(0)=1 and y'(0) = 1 (take h = 0.5)

(3+5)

6. a) How can you obtain numerical solution of a partial differential equation? Explain. **(3)**

   b) The steady-state two-dimensional heat-flow in a metal plate is defined by $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$. Given the boundary conditions as shown in figure below, find the temperature at interior points T_1, T_2, T_3 and T_4. **(5)**



7. Write an algorithm and C-program code to solve non-linear equation using Newton's method. Your program should read an initial guess from keyboard and display the followings if the solution is obtained: **(5+7)**

   - Estimated root of the equation
   - Functional value at calculated root
   - Required number of iterations

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(Numerical Method) — Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
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Attempt all questions:

1. What is bracketing and non-bracketing method? Explain with the help of example. Estimate a real root of following nonlinear equation using bisection method correct up to two significant figures

$$x^2 sinx + e^{-x} = 3.$$ **(3+5)**

2. Define interpolation. Find the functional value at x = 3.6 from the following data using forward difference table.

x	2	2.5	3	3.5	4	4.5
f(x)	1.43	1.03	0.76	0.6	0.48	0.39

(2+6)

3. Derive Simpson's 1/3 rule to evaluate numerical integration. Using this formula evaluate $\int_{0.2}^{1.2}(x^2 + lnx - sinx)dx$. [Take h = 0.1] **(4+4)**

4. What is pivoting? Why is it necessary? Explain. Solve the following set of equations using Gauss elimination or Gauss Seidel method.

$$x_1 + 10x_2 + x_3 = 24$$
$$10x_1 + x_2 + x_3 = 15$$
$$x_1 + x_2 + 10x_3 = 33$$

(3+5)

5. Compare Euler's method with Heun's method for solving differential equation. Obtain y(1.5) from given differential equation using Runge-Kutta 4th order method.
$\frac{dy}{dx} + 2x^2y = 1$ with y(1) = 0 (take h = 0.25) **(4+4)**

OR

Solve the following boundary value problem using shooting method.
$\frac{d^2y}{dx^2} - 2x^2y = 1$, with y(0) = 1 and y(1) = 1 [Take h = 0.5]. **(8)**

6. Solve the equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 3x^2y$ over the square domain $0 \leq x \leq 1.5$ and $0 \leq y \leq 1.5$ with *f=0* on the boundary [Take h = 0.5]. **(8)**

7. Write an algorithm and C-program to approximate the functional value at any given x from given n no. of data using Lagrange's interpolation. **(5+7)**

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(Numerical Method) — Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

1. How is the bisection method convergent to a root of an equation? Apply the bisection method to find a root of the equation
$$x\tan x - 1 = 0$$ **(3+5)**
2. Define interpolation. Find the Lagrange interpolation polynomial to fit the following data. Estimate the value

i	0	1	2	3
x_i	0	1	2	3
e^{x_i}	0	1.7183	6.3891	19.0855

of $e^{1.9}$ **(1+6+1)**

3. Derive Simpson's 1/3 rule to evaluate numerical integration. Using this formula evaluate $\int_0^2 \left(e^{x^2} - 1\right) dx$ with n = 8. **(4+4)**
4. What do you mean by ill-conditioned systems? Solve the following system using Dolittle LU decomposition method.
$$3x_1 + 2x_2 + x_3 = 24$$
$$2x_1 + 3x_2 + 2x_3 = 14$$
$$x_1 + 2x_2 + 3x_3 = 14$$
(2+6)
5. Solve the following boundary value problem using shooting method.
$\frac{d^2y}{dx^2} - 2x^2y = 1$, with y(0) = 1 and y(1) = 1 [Take h = 0.5]. **(8)**

6. Write the finite difference formula for solving Poisson's equation. Hence solve the Poisson's equation
$$\nabla^2 f = 2x^2y^2$$
over the domain $0 \le x \le 3$ and $0 \le y \le 3$ with *f = 0* on the boundary and h = 1. **(1+7)**

7. Write an algorithm and a C-program for the fixed point iteration method to find the roots of non-linear equation. **(4+8)**

OR

Write an algorithm and a C-program for the Lagrange's interpolation to approximate the functional value at any given x from given n data. **(4+8)**

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Computer Science and Information Technology (CSc. 204) | Pass Marks: 24

(Numerical Method) | Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
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Attempt all questions:

1. What are the sources of errors? Discuss various types of errors. Find the roots of the equation $x^2 + 5.6x - 10 = 0$ by trial and error method up to 4 significant digits. (1+3+4)
2. Describe Newton's method and its convergence. Find the root of equation $f(x) = e^x - 4x^2 = 0$ using Newton method up to 5 decimal places. (4+4)
3. What do you mean by interpolation and approximation? Use Lagrange interpolation to estimate the value of f(0.6) from the following table of values. (2+6)

x	0.4	0.5	0.7	0.8
f(x)	-0.916	-0.693	-0.357	-0.223

4. Using Newton's divided difference interpolating polynomial estimate the value of f(x) at x = 2.25 for the function defined as

x	0.5	0.2	1.4	2.2	3.0
f(x)	-10.25	-3.768	5.976	28.972	79.0

5. Write algorithm for Gauss- Seidel method for solving the system of linear equations. Also solve the following system of linear equations using that method. (4+4)

$$10x_1 + x_2 + x_3 = 12$$
$$x_1 + 10x_2 - x_3 = 10$$
$$x_1 - 2x_2 + 10x_3 = 9$$

6. What do you understand by the partial differential equation? Illustrate it with practical example and derive difference equation. (8)

OR

Find the solution of following differential equations using Taylor series method.
$y' = (x^3 + xy^2)e^{(-x)}$, $y(0) = 1$, to find y at x = 0.1, 0.2, 0.3.

7. Write an algorithm and program for computer to obtain the solution of differential equation using Runge-Kutta Method. {5+7)